Bayesian estimation of blur and noise in remote sensing imaging

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We propose a Bayesian approach to estimate the parameters of both blur and noise in remote sensing images. The goal is to infer the modulation transfer function (MTF) related to each observation, including atmospheric effects, optical blur, uniform motion and pixel-level sampling. The MTF is modeled by a real-valued parametric function with a small number of parameters. The noise is assumed to be an additive, white Gaussian process. Both blur and noise processes are supposed to be stationary. To constrain this ill-posed inverse problem, the unknown scene is modeled by a scale-invariant stochastic process governed by a fractal exponent and a global energy term. The main novelty consists of treating all parameters as random variables whose mean is estimated within a fully Bayesian framework. The chosen approach can be summarized as the computation of the mean posterior marginal related to useful parameters only. This requires integrating the joint probability density function (PDF) with respect to all the nuisance parameters, which is achieved through Laplace approximations. In this chapter we present two approaches; the former is straightforward, and the latter leads to a more efficient, simplified and optimized estimation algorithm. In addition, we investigate methods of uncertainty estimation and model assessment, in order to validate our approach on real images and to propose further improvements.

7.1 Introduction

In Earth observation, sometimes there is no other data about the scene of interest but a single picture, usually corrupted by blur and noise. In such a case, how is it possible to recover a good quality image of this scene? This difficult, under-determined problem is known as blind deconvolution. It covers many applications, ranging from astronomy and remote sensing to microscopy. However, we will restrict ourselves to satellite or aerial imaging, and we will mostly focus on natural images (as opposed to images of man-made structures). Moreover, our main interest will be in the degradation model, not in the image itself; we will assume that existing non-blind deblurring techniques could be used efficiently if we are able to provide both blur and noise models accurately enough. Numerous image deblurring algorithms have been developed over the past ten years [1, 2] but they are generally non-blind and therefore require a good knowledge of both blur and noise in order to work properly.

The large number of unknown variables, as well as the multiplicity and instability of solutions that are compatible with the observed image, make this problem particularly difficult to solve. It is twice ill posed, since both the image and the blur are unknown. Much effort has been put into finding satisfactory solutions in the past twenty years. Despite the apparent complexity of the task, people have managed to come up with interesting results by different means. Most methods require to constrain both image and blur in different ways. However, such constraints are often application-specific and could not be used in remote sensing. We will show in the next subsection that despite the rich literature in this domain, we had to develop a new method to be able to process satellite images effectively.

In the following, the same blur is denoted by either PSF or MTF, which respectively refer to the point spread function in the image space, and the modulation transfer function in the frequency space. The MTF is the modulus of the Fourier transform of the PSF.

7.1.1 Blind deconvolution: state of the art

There is no single way to classify the existing blind deconvolution methods. Several grouping schemes could be used depending on the application, the noise statistics, the blur kernel parametrization and the proposed estimation method. Reviews can be found in [3, 4]. Here we will use a classification based on these reviews.

7.1.1.1 Independent blur identification and deblurring

Let us first consider the group of methods that treat the blur estimation and the image restoration independently. There are a few techniques that are designed to identify the blur directly from an image, for instance from known scenes (such as man-made targets for instance) or through very strong prior knowledge, such as in Astronomy where stars are supposed to be points before being blurred. Therefore,
the PSF can be determined by deconvolving the observed image by the underlying object. Anyway, no such object is usually visible in satellite images. Weaker priors could be used to solve the problem in more general cases, for instance in X-ray imaging when some knowledge of the object contours are available [5]. Even in remote sensing, one could think of using geometric features such as straight roads or sea shores, though such features are not always encountered and seldom exhibit more than a few orientations, which impedes an accurate parameter estimation in all directions.

In some special cases, where camera motion or out-of-focus are considered, some methods can be used [6, 7, 8] that take advantage of the zeros in the observed image spectrum, related to zeros in the transfer function. The problem therefore reduces to locating these zeros, which can be tricky because of the noise. MTFs encountered in remote sensing rarely exhibit enough zeros to allow for a clear identification.

Some authors have used autoregressive models to represent the unknown image, combined with the blur and noise model to form a so-called autoregressive moving average process. In this framework, the model parameters can be simultaneously identified [9]. The estimation could be performed through expectation-maximization [10] or generalized cross-validation [11]. Other methods use the image bispectrum [7] to estimate the phase of the transfer function, but they suffer from instabilities.

The drawback of such methods is the lack of realism of the underlying scene model, which is assumed to be linear and stationary, without even capturing the scale invariance properties proper to natural images. When the scene contains sharp edges or spatially varying textures these models fail to estimate the transfer function correctly. This kind of global approach, including both scene and blur in the model, can succeed only if both scene and blur are modeled accurately.

Recently, new promising approaches have been developed. In the field of neural networks, the technique described in [12] was applied to satellite images to measure a parametric MTF from random images. However such a method requires a computationally expensive training step prior to the estimation, and it is unclear how well it will perform when the training dataset is limited to the observed image. Other approaches take advantage of the availability of at least two images of the same scene at different resolutions (i.e. 1m and 2.5m) to measure the ratio between low and high resolution MTFs [13]. Our approach relies upon a rather different theory and is worth developing mainly because it only needs a single input image.

7.1.1.2 Joint blur estimation and deblurring

The second group of methods aims at the restoration of the corrupted image and provides the blur function as a by-product. If any of these methods were able to solve our problem, we would stop here. Let us review them quickly to understand why they are not well-suited to remote sensing imagery.

Apart from a few theoretical findings, mathematically interesting but of little interest in practice (e.g. the zero-sheet separation [14]) because of unrealistic assumptions, such as special blur kernels or noise-free images, several methods proved to be useful in some special cases.

A first class of algorithms attempt to estimate the blur kernel pixelwise, assuming a small spatial support. The PSF pixels can be considered as random variables within a probabilistic framework. One will typically try to maximize the probability of both image and blur, which can be achieved through minimizing an energy term, usually non quadratic. The different constraints on this PSF (such as positivity, normalization and support for instance) can be embedded in properly designed prior probability densities. They can alternatively be introduced in an artificial way during the optimization [15] but without any proof of convergence. They can also be enforced during the optimization process via projections, penalty terms, reparametrization or even by doing constrained optimization. The energy term will then be minimized alternately w.r.t. the image and the PSF, thus forming the series of iterative blind deconvolution (IBD) algorithms.

Such techniques are proposed in [16] and [17] where constraints are enforced in both image and frequency spaces. Stability is not guaranteed. In [18] a non-quadric penalty term is used on both image and blur, which allows for edge-preserving regularization of the solutions. This is obviously better suited to boxcar PSFs than smoother, more regular and realistic ones. A similar approach is described in [19] in microwave imaging. Although smoother blur kernels can be recovered, such a method is sensitive to the prior choice; usually the prior is defined according to the physics of the problem, and must be close enough to the expected solution. Sometimes the regularization is achieved by stopping the optimization algorithm [20, 21] and projections are used instead of priors. In [22] the constraints are strictly enforced through PSF reparametrization.

Other methods different from IBD exist [3], and seem to converge more rapidly. However, the noise is not taken into account explicitly during the inversion process, therefore the algorithm is stopped before convergence to avoid noise amplification. Stochastic methods have also been proposed, such as in [23] where the authors use simulated annealing to perform the optimization.

More recently, a method based on sparse representations has been proposed [24]. It requires training on non-degraded images of the same type in order to determine a so-called sparsification kernel. Although this kind of iterative technique performs pretty well on noise-free images, it is unclear whether it is applicable at all in real cases where noise is present. In confocal microscopy, some authors propose to use the coefficients of a steerable pyramid to estimate the PSF [25].

Another class of methods assumes a parametric form of the blur kernel, which is a simple and elegant way to fulfill all the required conditions while taking into account the physics [4]. One may argue that this is too strong a constraint, and therefore does not allow for exotic, data-driven PSFs to be inferred. This is a weak argument: if the physics of the system is known, there is little chance to get odd-shaped blur kernels. Moreover, the theory of model selection would allow for the selection of the best parametrization among different solutions. In the worst case, model checking can help diagnose model failures, and design more appropriate blur models.

In any case, the parametric approach is far less complex and under-determined than the one relying on a pointwise estimate. The smaller number of parameters can be estimated using classical techniques such as maximum likelihood. For instance, in
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[26] an EM algorithm is used to perform such an estimation, using a scheme designed for astronomical images. Examples of such parametric methods can be found in [27] and [28] where the authors respectively use frequency space and \( L_1 \) regularization-based prior models of the unknown image. In [29] the blur is parametrized by the coefficients of a \( 2 \times 2 \) matrix; a nonstationary autoregressive image model is used and the estimation is performed within a Bayesian framework.

7.1.2 Constraining a difficult problem

Among all the previously developed algorithms, few can be applied to satellite or aerial images representing scenes chosen at random. Except for the inference from known targets, no method is actually able to provide an accurate estimate of the PSF from a single image, suitable for the remote sensing industry. Most of methods fail because they are designed to work in a different imaging domain (e.g. astronomy or microwave imaging). The constraints that help recover both blur and image in these cases are of little use on remote sensing scenes. The adequacy between model and image is essential. One of the key issues is to design a relevant image model.

A relevant model is not only useful for image restoration, but also helps discriminate between scene, blur and noise. This is why we choose to employ a power-law spectrum model; it is different enough from most physics-based MTFs as well as from white noise spectra. Existing approaches that only use models expressed in the image space (the ones exploiting low-order neighborhoods) fail to capture essential characteristics of natural scenes which exhibit details at all scales. The MTF can be merely seen as the attenuation of the spectrum; therefore spectral properties must be taken into account so that the MTF can be inferred. An obviously wrong spectrum model will systematically impede any attempt to recover it. Sometimes, neither image nor blur models are used at all, hoping that the constraints alone will manage to separate out the blur from the unknown scene. Commonly used constraints are insufficient in satellite imaging, hence our choice of a model-based approach.

Another key point is the reduction of the number of unknown variables. As stated previously, PSF or MTF parametrization help constrain the problem very efficiently. Thus, recovering the blur amounts to estimating a reduced number of parameters instead of determining the dozens or even hundreds of pixels of a discrete blur kernel. We choose to use parametric functions for both MTF and scene spectrum, so that we keep a pretty small parameter count. This will also help integrate out the unwanted variables and analyze the remaining ones easily, as it will be shown in the following sections. No doubt there are enough reasons to reduce the dimensionality of such an inference problem. As long as we do not lose useful information by doing so, we choose to start with the shortest description models; model checking shall tell us if we have to increase the dimensionality and/or change the parametrization.

7.1.3 The Bayesian viewpoint

Within a Bayesian framework [30], the model parameters are all assumed to be random variables. To solve a particular problem, one generally seeks to compute the posterior PDF of the variables of interest. It is posterior in the sense that it is conditioned upon all the observations. In this work, we focus on the determination of the mean of this posterior PDF.

However, the users of such probabilistic approaches sometimes do not try to remove the uninteresting variables from the PDF, to simplify the calculations. Though in a purely Bayesian approach, one must integrate over all other variables (also called nuisance variables) [29, 31, 32], the ones that are neither the parameters to be estimated, nor the observations. This step, also known as marginalization, can prove rather tricky in some cases. It is ignored most of the time for that very reason. Nonetheless, we will try to comply to the original methodology and integrate out the image model parameters, since we aim at the inference of the blur and noise parameters only. A few approximations will be required to achieve this integration in a computationally efficient way.

7.2 The forward model

In this section, we describe a generative model that is assumed to be a sufficient description of how the spectrum of an observed image is formed from an original, natural scene. Even though its simplicity prevents us from using it to deblur images efficiently, it provides a precise enough modeling of both degradations and original scene power spectrum to allow for a blur/noise/signal separation.

7.2.1 Modeling the natural scene using fractals

Natural scenes, assumed to be defined on a continuous bounded support included in \( \mathbb{R}^2 \), are supposed to follow a fractal model [33, 34]. This comes from the fact that the power spectrum of most images of natural landscapes (incidentally, it is also true with European cities) exhibits scale invariance properties [35]. Natural phenomena are statistically scale-invariant, which means that rescaling objects does not affect the measured statistics. This can also be called self-similarity: a scene resembles a scaled version of itself, or even a subset of itself – of course from a statistical viewpoint. The stochastic approach is then a convenient way to model these properties. Up to a multiplicative factor, the statistic regarded as a function of spatial variables does not change, whereas the transformed object can exhibit noticeable differences. Commonly used statistics are the expectations of autocorrelation functions and power spectra, depending on the working space. Let \( x \) and \( y \) denote the image space coordinates, and \( u \) and \( v \) the normalized spatial frequencies and \( r = \sqrt{u^2 + v^2} \) the radial frequency. The normalization is done w.r.t. the Nyquist frequency [36], so that \((u, v) \in [-1/2, 1/2]^2 \).

Let \( S \) be the scene, such that \( S_{xy} \in \mathbb{R}^+ \), defined over the bounded domain \( \Omega \subset \mathbb{R}^2 \),
and \( \mathcal{F} \), the Fourier transform; then our model is

\[
\mathcal{F}[\hat{S}]_{uv} = G_{\text{st}} w_0 r^{-q}
\]

(7.1)

where \( G \) is a Gaussian stationary process (Brownian motion) \(^{[34]}\) of marginal variance \( 1 \), and \( w_0 \) and \( q \) are respectively the energy and fractal exponent of the spectrum model. The mean power spectrum is the variance of the process and is equal to \( w_0^2 r^{-2q} \), which is an obviously scale-invariant power-law function.

There is another important property that somehow has to be accounted for, namely the nonstationarity. Scenes obviously exhibit a high spatial variability, since they are made of random arrangements of textured patches, sharp edges and sometimes almost constant areas. A simple way to take into account this variability is to define the process \( G \) in the image space, as the product between an uncorrelated Gaussian process of stationary variance 1 denoted by \( \mathcal{C} \), and a variable field denoted by \( L \), a real-valued function defined over \( \Omega \), possibly scale invariant but whose properties do not really matter for the studied problem. Thus, we set

\[
G_{uv} = \mathcal{F}[\mathcal{C} \times L]_{uv}
\]

(7.2)

This has no direct consequences on the mean power spectrum.

This model is well suited to natural images \(^{[37]}\) non-corrupted by blur or noise. It is also very convenient for remote sensing and, surprisingly enough, for urban areas. We performed experiments on a set of images representing different scenes (country, industrial areas, city centers, etc.) and found a good fit between this model and the observations. We plotted \( \log E_s(\{\mathcal{F}[\hat{S}]_r^2\}) \) as a function of \( r \), where \( E_s \) is the mean energy within a frequency band of width \( \delta r \) around the radial frequency \( r \). In order to avoid the effects of blur and noise, which corrupt any real image obtained with a sensor through an optical system, the image is subsampled by a factor ranging from 4 to 8, depending on the overall optical quality of the system. This also helps average out the noise. The subsampling is achieved in the frequency space by a sharp bandpass function. An example computed from aerial images is provided in Fig. 7.1.

Numerous experiments have shown that on real scenes \( q \) is between 0.9 and 1.5, which means that \( S \) can be modeled by a fractional Brownian motion \(^{[33]}\) in 2 dimensions.

### 7.2.2 Understanding the image formation

In this section, we briefly explain how to form a discrete image from a known scene defined over a continuous support. The whole physics-based process can be mathematically summarized by a simple canonical transform consisting of blurring, sampling and noise addition \(^{[36,38]}\).

#### 7.2.2.1 Atmospheric, optical and sensor MTF modeling

We assume a shift-invariant, convolutional blur, which can be best described by a MTF (no transfer function phase recovery will be attempted here) in the frequency space. Its parameters are denoted by \( \alpha \). This enables us to take into account the physics of light scattering, diffraction, pixel integration and other non-optical phenomena that affect the overall blur. Since all the different phenomena are modeled by successive convolutions, the overall MTF is the product of all the individual terms:

\[
\text{MTF} = \text{MTF}_{\text{atm}} \times \text{MTF}_{\text{opt}} \times \text{MTF}_{\text{mot}} \times \text{MTF}_{\text{sen}}
\]

(7.3)

which respectively summarize the atmospheric, optical system, motion and the sensor contributions.

The atmosphere contributes through two major phenomena. Turbulence, which heavily distorts the wavefront, has random effects on both phase and magnitude of the transfer function. However for long enough exposures its contribution merely...
The forward model consists of a deterministic space-invariant PSF, equivalent to a MTF expressed as \( \exp(-\kappa r^2/2) \) [39]. On the other hand, aerosols contribute to light scattering and attenuation. The effect on the MTF can be modeled by \( \exp(-\kappa (r/r_e)^2) \) for \( r \) below the cut-off frequency \( r_e \) (and a constant above).

The optical system has a limited aperture which causes a diffraction blur, whose MTF is given by the autocorrelation of the pupil function [40]. It is mostly characterized by a cut-off frequency which is an increasing function of the aperture. Its expression is rather complex and involves Bessel functions. Optical aberrations (which include defocus and spherical aberrations) can be expressed as additive perturbations to this MTF. In general, they are computed numerically.

The third term groups all the motion-related blurs: shift and vibrations, which are common problems aboard satellites. It can be approximated by an expression involving cardinal sine functions related to combinations of uniform motions along one or two directions. We assume a motion blur without acceleration and only try to recover a minimum-phase blur. This is due to the chosen Gaussian image model and related second-order statistics that are insensitive to phase shifts.

Finally, the sensor also contributes to the blur, mostly because the irradiance is integrated over pixels, but also because of charge diffusion or transfer problems between adjacent pixels. For a matrix sensor with pixel size \( p \) and sampling grid size \( \Delta \) (allowing for gaps between pixels) the integration MTF is given by

\[
\langle \text{MTF}_{\text{int}} \rangle_{uv} = \text{sinc} \left( \frac{\pi u p}{\Delta} \right) \text{sinc} \left( \frac{\pi v p}{\Delta} \right)
\]

whereas the diffusion process can be modeled by a Gaussian \( e^{-\kappa r^2} \).

All these imaging-chain transfer functions can be gathered into a single term denoted by \( F \), and we leave the integration term out: \( \text{MTF} = \text{MTF}_{\text{int}} \times F \). One can start with a Gaussian function:

\[
F_{uv} = e^{-\left(\alpha u^2 + \alpha v^2\right)}
\]

arguing that the convolution of the many blurs mentioned above is a nearly Gaussian function. This model could be further refined via a model selection step if it proves to be inaccurate, or if some of the physical characteristics of the system are known.

To avoid problems related to spectral overlapping due to sampling in the image space, we use a bandpass boxcar function, \( \Pi_k \), equal to 1 over the useful frequency range. Then we get:

\[
\text{MTF} = \text{MTF}_0 \times F \quad \text{where} \quad \text{MTF}_0 = \Pi_k \times \text{MTF}_{\text{int}}
\]

7.2.2.2 Sampling and sensor noise

We denote the observed image by \( Y \). The observed pixel values are obtained by sampling the deterministic blurred scene on a regular square grid (pixel size \( p \)), and adding a stochastic noise denoted by \( e \). The pixel coordinates \( i \) and \( j \) range from 0 to \( N-1 \) (square image of \( N \times N \) pixels).

\[
Y_{ij} = (S \ast \mathcal{F}^{-1}[\text{MTF}])_{\mu(pj)} + e_{ij}
\]

### 7.3 Bayesian estimation: invert the forward model

The goal is to compute the mode of the posterior PDF of the parameters of interest \( \alpha \) and \( \sigma \), i.e., \( P(\alpha | \tilde{Y}) \) and \( P(\sigma | \tilde{Y}) \). This involves two steps: apply Bayes rule to invert the forward model, and marginalize or integrate w.r.t. the prior model parameters \( w_0 \) and \( q \) which are nuisance variables. This can be summarized as:

\[
P(\alpha | \tilde{Y}) \propto P(\alpha) \int P(\tilde{Y} | w_0, q, \alpha, \sigma) P(w_0, q | \sigma) d w_0 dq d \sigma
\]
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In the following, we will assume flat priors on all parameters, so that the equation above amounts to integrating the likelihood defined by (7.11). Therefore the main problem to solve states as follows (solving it also provides a solution for $\sigma$):

$$\hat{\alpha} = \arg\max_{\alpha} \int P(\hat{Y} \mid w_0, q, \alpha, \sigma) dW_0 d\sigma$$

(7.13)

We can write similar equations for $P(\sigma \mid \hat{Y})$ and $\hat{\sigma}$.

### 7.3.1 Marginalization and related approximations

Computing the integral (7.12), also known as marginalization, is intractable in general. We propose to use a Laplace approximation [41], which involves a quadratic approximation of the -log likelihood or energy $U$ around its optimum in $\theta = (w_0, q, \alpha, \sigma)$:

$$U(w_0, q, \alpha, \sigma, \hat{Y}) = -\log P(\hat{Y} \mid w_0, q, \alpha, \sigma) = \sum_{k} \log (2\pi \omega_k^2) + \frac{|\hat{Y}_k|^2}{2\omega_k^2}$$

(7.14)

$$\simeq U(\hat{\theta}, \alpha, \hat{Y}) + \frac{1}{2} (\theta - \hat{\theta})^T [\Sigma_{\theta \theta}]^{-1} (\theta - \hat{\theta})$$

(7.15)

where the $\omega_k$ are given by (7.11). Therefore we have [41]:

$$\int P(\hat{Y} \mid \theta, \alpha, \sigma) d\theta \simeq \sqrt{2\pi \Sigma_{\theta \theta}} e^{-\frac{1}{2} (\theta - \hat{\theta})^T [\Sigma_{\theta \theta}]^{-1} (\theta - \hat{\theta})}$$

(7.16)

The problem (7.13) can be reformulated as the minimization of a new energy:

$$\hat{\alpha} = \arg\min_{\alpha} \left[ U(\hat{\theta}, \alpha, \hat{Y}) - \frac{1}{2} \log |\Sigma_{\theta \theta}| \right]$$

(7.17)

In this equation, the variations of the log-determinant term w.r.t. $\alpha$ can be neglected w.r.t. the variations of the optimal energy $U(\hat{\theta})$. We have checked this assumption experimentally. Although the determinant can be explicitly computed using second derivatives of $U$, and differentiated w.r.t. $\alpha$, the drop-off in the computational efficiency was not justified by the enhancement of the estimation results.

Finally, we can write the search for $\alpha$ as two nested optimizations:

$$\hat{\theta} = \arg\min_{w_0, q, \sigma} U(w_0, q, \alpha, \sigma, \hat{Y})$$

(7.18)

which leads to an algorithm with two nested loops, which is fundamentally different from the joint optimization w.r.t. MTF, prior model and noise parameters. As a consequence, the resulting method is more stable than usual joint methods and therefore better copes with such nonlinear cases. As shown in [32], it is necessary to integrate the likelihood w.r.t. the nuisance parameters to get a robust estimate, i.e. to compute the marginalized likelihood instead of the joint likelihood.

### 7.3.2 A natural parameter estimation algorithm (BLINDE)

In this section we present an algorithm called BLINDE (BLInD DEconvolution) based on the approach described above, initially described in [42] and successfully applied to simulated remote sensing imagery (SPOT 5 and Pleiades satellites) provided by the French Space Agency (CNES). The general approach is patented1.

#### 7.3.2.1 Noise variance marginalization

The noise variance and the prior model parameters are optimized separately in this approach. We assume that the energy coming from the signal is negligible w.r.t. the contribution of the noise in the highest frequencies, i.e. $\sigma_e \in [1/2, \sqrt{2}]$. Therefore it is estimated from the power spectrum in these frequencies, after masking the bands $u = 0$ and $v = 0$ as well as the corners such that $r > r'$, in order to avoid artifacts related to bad rows or columns, stripes, interferences and the finite size of the image (boundary effects). See Fig. 7.2 for an illustration. Except for such artifacts, the highest frequencies of $\hat{Y}$ contain only noise as long as the image is blurred enough, or at least correctly sampled in all directions. Other methods could be used for noise variance estimation [43, 44].

![FIGURE 7.2: Noise estimation in the frequency space by measuring the energy in the spectral ring $r' < r < r''$ (assumed to contain only white Gaussian noise).](image)

#### 7.3.2.2 Prior model marginalization

Once the noise standard deviation has been determined, we have to optimize $U$ w.r.t. $w_0$ and $q$ while $\alpha$ is fixed. This can be achieved through various gradient-based methods (conjugate gradient for instance). We give here the derivatives of $U$ defined by Eqn. (7.14) involved in the minimization:

$$\frac{\partial U}{\partial w_0} = -\sum_k \frac{1}{\omega_k^2} \left(1 - \frac{|\hat{Y}_k|^2}{2\omega_k^2}\right) \frac{\partial \omega_k^2}{\partial \theta}$$

(7.19)

where $\frac{\partial \omega_k^2}{\partial w_0} = 2w_0^2 \text{MTF}_w^2$ and $\frac{\partial \omega_k^2}{\partial q} = -2\log(w_0) w_0^2 r_0^{2\omega} \text{MTF}_w^2$ (7.20)

1Patents: USA #28540234162 (2002); Europe #02774858.1 (2002); France #0101089 (2001).
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To accelerate this optimization step (it is critical for the overall computational efficiency since this has to be done every time the MTF parameters are updated), we used a linear regression technique which has the advantage to provide a closed-form solution. Recall the log plots from Fig. 7.1. For a non-degraded image, the log radial power spectrum denoted by $y$ is a linear function of the log radial frequency denoted by $x$, such that $y = w_0 - q x$. The trick consists of “deconvolving” the mean radial power spectrum using the current estimate of the MTF and noise variance:

$$ y_i = \log E_x [\hat{T}_r (\hat{Y}) \text{MTF}^{-1}] \quad x_i = \log r_i $$  \hspace{1cm} (7.21)

where $r_0$ is a soft thresholding function and $E$ denotes the average over the angle $\theta$ such that $(r, \theta)$ is the polar representation of $(u, v)$. It is applied to both real and imaginary parts of the Fourier coefficients $\hat{Y}$ to avoid noise amplification when dividing by the MTF. Frequency bands yielding too small energies are not taken into account in the regression. The variances related to each $y_i$ are set proportional to $x_i$ assuming these variables have a $\chi^2$ distribution since they are obtained by summing up a large number of Gaussian coefficients.

This method is not guaranteed to provide accurate estimates, therefore it can be used as a quick initialization of an iterative method. This way, only a few steps of conjugate gradient (less than 5) are needed.

7.3.2.3 The algorithm

Let us focus on the highest level optimization procedure, seeking the value of $\alpha$ that minimizes the energy $U(\hat{\alpha})$ where $\hat{\alpha}$ is computed as explained above. If we use a gradient-based method, the first derivatives are needed. Since $\hat{\alpha}$ satisfies $\partial U / \partial \alpha = 0$, even though $\hat{\alpha}$ depends on $\alpha$, we have:

$$ \frac{\partial U(\hat{\alpha})}{\partial \alpha} = \frac{\partial U(\hat{\alpha}, \hat{Y})}{\partial \alpha} = \sum_{i} \frac{1}{\hat{v}_i \hat{u}_i} \left( 1 - \frac{\hat{Y}_i^2}{2 \hat{v}_i^2} \right) \frac{\partial \hat{v}_i^2}{\partial \alpha} \bigg|_{\alpha} $$  \hspace{1cm} (7.22)

where

$$ \frac{\partial \hat{v}_i^2}{\partial \alpha} = w_0^2 r_{0w}^{-2\hat{v}_i} \text{MTF}^2 \frac{\partial F_{\hat{v}_i}}{\partial \alpha} $$  \hspace{1cm} (7.23)

For a Gaussian blur kernel $F$ and a bidimensional vector $\alpha$ we have:

$$ \frac{\partial \hat{v}_i^2}{\partial \alpha} = -2 w_0^2 r_{0w}^{-2\hat{v}_i} \text{MTF}^2 \hat{v}_i \cdot \hat{v}_i $$  \hspace{1cm} (7.24)

The optimization is performed using a conjugate gradient (CG) method adapted to non-quadratic functions, which uses line minimizations (involving gradient computation and projection along the search direction) as well as an update iteration for the search direction, once the line minimization has converged. Details can be found in [45]. The convergence properties are well-known and beyond the scope of this paper. It is applied under particular conditions as approximations are made in order to simplify the marginalization-related computations. However, experimental studies have shown a good stability, on simulations from the Gaussian image model with known parameter values, as well as on artificially degraded remote sensing images. No more than 30 iterations are needed when initialized with $\alpha = 0$.

- **Initialization.** The variance of the noise is estimated from $\hat{Y}$, using only very high frequencies $r > 1/2$.
- **(1) Inner optimization loop:**
  - Initialization: linear regression using the data from Eqn. (7.21).
  - Iterative optimization: conjugate gradient using the derivatives of Eqns. (7.19)-(7.20) to get $\hat{\alpha}$.
- **Evaluate $U(\hat{\alpha})$ and its derivatives w.r.t. $\alpha$ using Eqns. (7.22)-(7.23).**
- **Update $\alpha$.** Update the CG direction. A line minimization step is required and the inner loop has to be run for each value of $\alpha$ involved in this step.
- **Go to step (1) until convergence is reached.**

This method is fundamentally different from alternate minimization schemes usually employed, since we have to optimize w.r.t. $\alpha$ each time $\hat{\alpha}$ is changed.

7.3.3 Why use a simplified model?

The nonlinearity of the previous approach and the high number of data points (causing the method to slow down when increasing the image size) motivate the choice of a new, simplified model. This model enables us to reduce the data so that the complexity of the iterative estimation procedure does not depend on the image size any longer (except for an initial FFT). It should also reduce the dependence between the observed variables.

We reformulate the observation model through a binning process. The spectrum coefficients $Y_0$ are transformed into $M$ new variables $z_0$. Let each $z_0$ be the mean squared magnitude of $\hat{Y}$ over the $n$-th frequency bin, defined by a square area of size $B \times B$ around the frequential coordinates $(k_0^n, l_0^n)$.

$$ z_0 = \frac{1}{B} \sum_{k=-B}^{B} \sum_{l=-B}^{B} |\hat{Y}_{u,v}|^2 $$  \hspace{1cm} (7.25)

This way, if the bins are large enough the dependencies can be substantially reduced and we can assume independent observations $\{z_0\}$. This also reduces the dimensionality of the problem and the computational complexity. The bin size shall be as large as possible to minimize the number of data points, without compromising the assumed stationarity of the mean power spectrum. We will not discuss the optimal choice of the bin size in this paper; refer to the model assessment section for details about how to validate the proposed model in general. The binning and the selected spatial frequency domain are illustrated by Fig. 7.3.

Sums of independent Gaussian variables have a $\chi^2$ distribution. Despite the weak correlation between Fourier coefficients (due to the convolution by $F[s]$) we use an
Bayesian estimation: invert the forward model

Figure 7.3: Dark squares: spatial frequency binning. Shaded area: radial frequency clipping and removal of frequencies close to \( a = 0 \) and \( v = 0 \), to ensure the validity of the model while avoiding various artifacts (bad rows or columns, image periodization, structured noise).

The independence assumption, so that the variance of each \( z_n \) can be approximated by \( 2z_n \). Denoting the mean of \( z_n \) by \( \mu_n \), we have:

\[
P(z | w_0, q, \alpha, \sigma) = \prod_n \mathcal{A}_n^1 \left( \mu_n, \sigma_n^2 \right)
\]

\[\begin{align*}
\mu_n &= \frac{w_0^2 r_n^{-2\theta} \text{MTF}_n^2 + \sigma^2}{2z_n} \\
\sigma_n^2 &= 2z_n
\end{align*}\]  

(7.26)

Here, \( r_n \) and MTF \( \text{MTF}_n \) denote the radial frequency and MTF averaged over the respective bin; they can be approximated by the values taken at the central spatial frequency of the bin, i.e., at \((k_0^2/N, \rho_0^2/N)\).

7.3.4 A simplified, optimized algorithm

The joint -log likelihood is now denoted by \( V \) and writes:

\[
V(w_0, q, \alpha, \sigma, z) = -\log P(z | w_0, q, \alpha, \sigma) - \sum_n \left( \frac{\mu_n - z_n}{4z_n} \right) + \text{const.} 
\]

(7.27)

The constant term is related to the normalization and depends only on \( z \) if the variance of \( z \) only depends on the observed data (we will show in the result section how to re-estimate this variance when needed). This yields a much simpler expression of the energy term than we had before. If we set \( \phi = (w_0, \sigma^2) \), we can see that the optimization problem is linear w.r.t. \( \phi \). Therefore there is a closed-form solution for the optimal \( w_0^\phi \) and \( \sigma^2 \) for each value of \( q \):

\[
\frac{\partial V}{\partial w_0} = \sum_n \left( \frac{\mu_n - z_n}{2z_n} \right) \frac{\partial \mu_n}{\partial w_0} 
\]

(7.28)

where \( \frac{\partial \mu_n}{\partial w_0} = r_n^{-2\theta} \text{MTF}_n^2 \) and \( \frac{\partial \mu_n}{\partial \sigma^2} = 1 \)

(7.29)

We have the linear system \((A(q)) \phi = c(q)\) where:

\[
A(q) = \begin{bmatrix}
\sum r_n^{-2\theta} \text{MTF}_n^2/z_n & \sum r_n^{-2\theta} \text{MTF}_n^2/z_n \\
\sum r_n^{-2\theta} \text{MTF}_n^2/z_n & \sum 1/z_n
\end{bmatrix}
\]

\[
c(q) = \begin{bmatrix}
\sum r_n^{-2\theta} \text{MTF}_n^2 \\
\sum r_n^{-2\theta} \text{MTF}_n^2 \\
\sum 1/z_n
\end{bmatrix}
\]

(7.30)

which yields \( \hat{\phi} = (A(q))^{-1} c(q) \).

A 1D line search can be performed to find the optimal value of \( q \). Since \( \hat{\phi} \) satisfies \( \partial V/\partial \phi = 0 \), even though \( \phi \) depends on \( q \), the derivative of \( V(\phi) \) w.r.t. \( q \) writes:

\[
\frac{\partial V(\hat{\phi}(q), q, \alpha, z)}{\partial q} = - \sum_n \left( \frac{\mu_n - z_n}{z_n} \right) \log(r_n) w_0^2 r_n^{-2\theta} \text{MTF}_n^2 | \hat{\phi}
\]

(7.31)

Thus, there is no need for an approximate method (such as the one based on linear regression of paragraph 7.3.2) or any multidimensional gradient descent technique. Moreover, the noise variance is marginalized as well, thus avoiding the initialization step in which it is estimated from the highest frequencies of the power spectrum.

Now we can focus on the estimation of the MTF parameters using this second approach. We keep exactly the same marginalization framework as in the previous approach (see section 7.3.1), with a different notation. Eqn. (7.18) remains valid (with \( V, z \) instead of \( U, \hat{V} \)). The derivatives w.r.t. \( \alpha \) are given by:

\[
\frac{\partial V(\hat{\theta}(\alpha), q, \alpha, z)}{\partial \alpha} = \frac{\partial V(\hat{\theta}(\theta, \alpha, z))}{\partial \alpha} = - \sum_n \left( \frac{\mu_n - z_n}{z_n} \right) \frac{\partial \mu_n}{\partial \alpha} | \hat{\phi}
\]

(7.32)

where \( \frac{\partial \mu_n}{\partial \alpha} = w_0^2 r_n^{-2\theta} \text{MTF}_n^2 | \theta \)

(7.33)

For a Gaussian blur, we have:

\[
\frac{\partial \mu_n}{\partial \alpha} = -2w_0^2 r_n^{-2\theta} \text{MTF}_n^2 | \mu_n^2 \psi_n^2
\]

(7.34)

The optimization algorithm relies again on a conjugate gradient method adapted to non-quadratic functions [45]. Regarding convergence, the remarks we made for the BLINDE algorithm still hold.

- (1) Inner optimization loop: Line optimization using the derivative (7.31) where \( w_0^\phi \) and \( \sigma^2 \) are replaced by their closed-form optimum using (7.30).
- Evaluate \( V(\hat{\theta}) \) and its derivatives w.r.t. \( \alpha \) using Eqns. (7.32)-(7.33).
- Update \( \alpha \). Update the CG direction. A line minimization step is required and the inner loop has to be run for each value of \( \alpha \) involved in this step.
- Go to step (1) until convergence is reached.

Other optimization methods could be used. Recently, we have developed a quasi-Newton scheme based on the second derivatives of the marginalized \( V \) w.r.t. \( \alpha \). It seems to perform better than the CG, since it does not require any accurate line minimization at each step in order to converge quickly. However, it might be more sensitive to the initialization because it involves an inversion of the Hessian matrix at each update step. Nonetheless, the problem can be stabilized by using a linear approximation of \( \mu \) when computing this matrix.
7.4 Possible improvements and further development

7.4.1 Computing uncertainties

The Bayesian formalism can be applied to perform inference instead of point estimation. It is possible to compute a Gaussian approximation to the posterior PDF (7.12), characterized not only by a mean, but also by a covariance matrix. The covariance provides a measure of the uncertainty related to the estimated parameters. If the PDF is proportional to $e^{-V(\theta)}$, a quadratic approximation is made around the optimum $\hat{\theta}$ of $V$ in which the second derivatives are the elements of the inverse covariance matrix. Marginalization w.r.t. prior model parameters amounts to keeping the entries of the covariance matrix related to the remaining variables, i.e. $\alpha$.

However we encountered difficulties in obtaining error estimates that were consistent with the ground truth. In our experiments (simulations of degraded images using predefined MTF parameters), 4 out of 5 true parameter values were out of the 99% confidence ellipses, meaning that the actual error was bigger than the predicted one. Which brings us to the conclusion that no matter how well uncertainties are computed for a particular model, they are correct only as long as the model provides an accurate description of the underlying scene. Even with an imprecise or inadequate model, computed uncertainties can be made arbitrarily small if a large enough number of data points are measured, thus mistakenly concluding that the inference is accurate. Only an appropriate model checking procedure could tell us if our modeling is flawed.

7.4.2 Model assessment and checking

We can distinguish between two main sources of estimation errors:

- **The lack of robustness of the input data $z$ of Eqn. (7.25).** The sum of squares statistic is actually inadequate for any non-Gaussian distribution, since it yields estimates that are quite sensitive to outliers. One can observe discrepancies between the assumed Gaussian model for Fourier coefficients and the actual data, occurring if there are unusual spikes at particular frequencies in the spectrum, or holes in the spectrum, due to artificial structures or a lack of features in the scene. These discrepancies could be better detected using wavelet transforms rather than the Fourier transform, since they can be strongly related to the nonstationarity property of the scene. Instead of using the sum of squares, one could use an order-$p$ moment (with $p=1$ for instance) as a robust estimator of the mean power spectrum.

- **The discrepancy between the assumed $1/f$ model and the actual spectral decay of the scene.** In practice, natural scenes of limited size are not perfectly fractal. One can expect unusual fast decays of the power spectrum in the highest frequencies, or the contrary. This can be seen as a curvature in the log-log plot, which inevitably leads to MTF parameter over- or under-estimation, depending on the sign of the curvature. Such behaviors are particularly difficult to detect. This can lead to indetermination: degenerate solutions that fit the data can be found, such that various prior model curvatures and MTF parameter values lead to the exact same predicted mean $\mu$ of Eqn. (7.26).

Model assessment can be performed in various ways; the simplest way to check the statistical validity of a model such as the one used in this work is through residual analysis. There are a variety of tests that can be run on the residuals (differences $z_\alpha - \mu_\alpha$) once the estimation has been done [31]. For instance the sum of squared, normalized residuals is supposed to have a $\chi^2$ distribution with $M - n$ degrees of freedom, where $n$ is the number of parameters, if the error model is Gaussian. Outliers, or random modeling errors, can typically be detected by such tests. However, epistemic errors (coming from an inappropriate model choice) could not be detected this way. Therefore simple statistical tests are not sufficient and one must resort to model comparison in order to select the best model, or the best combination of models.

Automatic model selection (or model averaging) can be performed in the Bayesian framework [31, 41], by adding a new variable acting as a model label and infer it (assuming we have a choice among a discrete set of models) or by adding new parameters to the model itself until some optimality condition is reached (relating for instance to the best compromise between model sparsity and prediction accuracy). In all cases, marginalization will have to be done in order to integrate out these extra variables. To summarize this, determining the posterior PDF (7.12) is still the goal, there are simply a few more variables to be integrated out. The higher computational complexity would certainly be justified by the increased robustness of the resulting algorithm.

7.4.3 Robustness-related improvements

Two enhancements can be proposed in a recursive way, in order to improve the overall robustness of the algorithm. First, the error estimates $e$ can be updated according to the estimated spectrum model, not to the observed values $\hat{z}$. Second, outliers can be identified and rejected, according to the statistical significance of the residual related to each data point (for instance, one could reject all points having a residual magnitude greater than $2\sigma$, which corresponds to less than 5% statistical significance). Finally, the estimation algorithm has to be run again. This is what was done in the experiments shown in the next section. Ideally, the whole process shall be repeated until convergence. This method should probably handle stochastic modeling errors efficiently, however it would be of little use in case of a systematic model mismatch (e.g. power spectrum curvature on a log-log scale).
7.5 Results

7.5.1 First method: BLINDE

The MTF parameter $\alpha$ of an isotropic Gaussian blur (Eqn. (7.5) with $\alpha_b = \alpha_t$) was computed from simulations performed on both an aerial image of the city of Amiens provided by IGN (French Geographic Institute), and an aerial image of the city of Nîmes provided by CNES (French Space Agency), with respective subsampling factors 8 and 3. Subsampling through a band-pass boxcar function in the frequency space was necessary in order to avoid artifacts coming from the MTF of the instrument (telescope, motion...). Blurred and noisy satellite image simulations are $512 \times 512$ and about $2.5\,m$ in resolution due to the subsampling factor. Figure 7.4 displays results obtained with the BLINDE algorithm, showing a good accuracy (better than 10%) on the MTF value at $r = 1/4$. The overall accuracy seems to increase with the blur size. The method was also directly applied to an area extracted from the real image of Amiens at $30\,cm$ resolution (see fig. 7.5). We found a realistic blur size close to the optimal sampling rate, but could not check it against any ground truth.

7.5.2 Second method

Experimental settings The MTF parameters $\alpha_b$ and $\alpha_t$ of a Gaussian blur of Eqn. (7.5) were estimated from simulations performed on subsampled Ikonos images (credit Space Imaging). We choose subsampling factors from 2 to 4 to avoid artifacts coming from the MTF of the instrument (telescope, motion...) or the lossy compression scheme. We assumed that after a frequency-space subsampling through a boxcar bandpass function these images exhibit a fractal behavior, assumption that can be checked by analyzing the power spectrum as shown on Fig. 7.10. We choose to use simulations (obtained by applying blur and noise to such subsampled images) because of the lack of availability of real images whose blur is perfectly calibrated.

Figures 7.6-7.9 display the 4 chosen scenes. For each one of the scenes, we show 3 examples of corrupted observations (small region of interest) for 3 blur parameter settings (denoted by a, b, c in the summary tables 7.1 and 7.2) corresponding to $\sigma^2 = 3$ and $\alpha_b, \alpha_t \in (10, 30)$. For each setting, a small region of the degraded image is shown (left subfigure). The transfer functions along the horizontal and vertical axes are plotted (respectively for $v = 0$ and $u = 0$, see center and right subfigures) for both true and estimated parameters. Thus the consequence of any parameter misestimation on the overall transfer function can be quickly visualized (systems designers are usually interested in MTF values at $u$ or $v$ equal to 1/4 or 1/2, which can be read from these graphs).

Tables 7.1 and 7.2 summarize all our experiments made on the 4 different scenes, from simulated blurred and noisy images at 2 different sizes (and 2 respective binnings $32 \times 32$ and $16 \times 16$), for 2 values of the noise variance $\sigma^2 \in \{3, 9\}$ and 4 combinations of MTF parameters (both isotropic and anisotropic) $(\alpha_b, \alpha_t) \in (10, 30)^2$ indexed by (a,b,c,...,h). The value of the MTF at 3 spatial frequencies ($u$ and $v$ equal to 1/4) are shown for both true and estimated parameter values. The difference between true and estimated values at these 3 points is proposed as a simple absolute error measure.

To analyze the residuals, we define the normalized chi-squared residual as $k = (\hat{x}^2 - M)/\sqrt{2M}$ where $M$ is a good approximation of the number of degrees of freedom ($M=435$ in the experiments and corresponds to a spectrum subdivision into $32 \times 16$ bins among which we keep the ones having $0.1 < r < 0.5$, $u \neq 0$ and $v \neq 0$). The rejected outlier rates $p$ and $k$ give an idea of the estimated model quality. However, there is no way to assess the appropriateness of the fractal model directly from the blurred images. This is why we also display the power spectra of the used images without degradation (and with an ideal box sampling in the frequency domain) on Fig. 7.10 using the same binning $z$ as in the second estimation method. This figure clearly shows that the fractal model is not perfect, since there are many outliers and also some significant systematic deviations. We notice that this is not directly related to the statistics of the underlying scene; unfortunately the images were obtained by a real optical system and sensor, and were processed in a way unknown to us.

Result discussion and comments Matangi (Fig. 7.6) is mostly made of natural features (coastlines and clouds) so the estimation is expected to be accurate since the image content is well described by a fractal model. Helliniko (Fig. 7.7) displays a complex series of man-made features, and it surprisingly provides the best results among the 4 scenes. Whether artificial constructions are located or not on underlying natural structures is far beyond the scope of this study; however it seems that these constructions obey the proposed power-law decay of the power spectrum, which enables the entire approach to clearly identify the blur parameters. Also, the spectral content is fairly rich as shown on Fig. 7.10 which undoubtedly helps estimate the parameters more accurately.

Goosenek (Fig. 7.8) and Stonehenge (Fig. 7.9), on the other hand, have a relatively poor spectrum and even some unwanted texture (which is obviously not self-similar), which could explain why our method does not perform so well on these images. We can notice that there is a systematic over-estimation of the MTF. Despite the apparent good match between data and model for the Gooseneck image on Fig. 7.10, the log-log spectrum of the $512 \times 512$ image is probably bent in such a way that the MTF is consistently overestimated. It appears that on these two scenes, there are more high frequencies than predicted by the model (or equivalently, less lower frequencies). This can be due to multiple reasons, including piecewise scale invariance intrinsic to the scene, and pre-processing artifacts we were not aware of (even a slight sharpening would be sufficient to explain this behavior).

As shown on the summary tables 7.1 and 7.2, normalized residuals are high enough to suggest that the spectrum model is not sufficient to model all kinds of scenes. These residuals are highest for man-made features (Helliniko buildings and Stonehenge fields). Outlier rejection helps reduce the residuals in some cases (as in Helliniko) but fails to solve the model mismatch problem in other cases (as in Stonehenge) especially when the spectral content is poor. In general, the number of outliers tends to increase with the image size, so we conclude that a subsampling factor 2 is
Results

insufficient to reduce the artifacts on the source images, except for Gooseneck where the best results are obtained from the highest resolution image. On real data however, one should expect better results when using a larger number of pixels.

In order to check the performance of the proposed method in a more accurate way, we recommend the use of real images (instead of simulations) whenever possible, with a good determination of their degradation model (by traditional means such as target imaging for instance).

\[ \hat{\alpha} = 30; \sigma = 5; \tilde{x} = 25.5; \tilde{y} = 5.3 \]

**FIGURE 7.4:** Images extracted from 512 × 512 simulations of blurred and noisy satellite images at 2.5m resolution (top: Amiens © IGN, bottom: Nîmes © CNES). The isotropic MTF (left) and PSF (right) are plotted along the horizontal axis \( \nu = 0 \). Dashed lines: estimated blur.

**FIGURE 7.5:** Region extracted from a real, blurred and noisy aerial image of Amiens © IGN, 30cm resolution. The estimated isotropic MTF (left) and PSF (right) are plotted in solid lines along the horizontal axis (respectively \( \nu = 0 \) and \( \eta = 0 \)). (we found \( \hat{\alpha} \approx 5 \).)
FIGURE 7.6: Subsampled image of Matangi © Space Imaging, 512 × 512, 4 m final resolution and small region of interest (ROI). For each experimental setting a,b,c refer to table 7.1 for the related parameter values, we show from left to right the degraded ROI and MTF plots along horizontal and vertical axes (resp. for u = 0 and v = 0). Dashed lines represent estimated MTFs.

FIGURE 7.7: Subsampled image of Helliniko © Space Imaging, 512 × 512, 4 m final resolution and small region of interest (ROI). For each experimental setting a,b,c refer to table 7.1 for the related parameter values, we show from left to right the degraded ROI and MTF plots along horizontal and vertical axes (resp. for u = 0 and v = 0). Dashed lines represent estimated MTFs.
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FIGURE 7.8: Subsampled image of Gooseneck © Space Imaging, 512×512, 4m final resolution and small region of interest (ROI). For each experimental setting a,b,c (refer to table 7.1 for the related parameter values), we show from left to right the degraded ROI, and MTF plots along horizontal and vertical axes (resp. for $u = 0$ and $v = 0$). Dashed lines represent estimated MTFs.

FIGURE 7.9: Subsampled image of Stonehenge © Space Imaging, 512×512, 4m final resolution and small region of interest (ROI). For each experimental setting a,b,c (refer to table 7.1 for the related parameter values), we show from left to right the degraded ROI, and MTF plots along horizontal and vertical axes (resp. for $u = 0$ and $v = 0$). Dashed lines represent estimated MTFs.
Bayesian estimation of blur and noise in remote sensing imaging

### FIGURE 7.10: Log-log plots showing the mean power spectra ($\chi$ variables defined by Eq. (7.25)) and related error bars for $r \in [0.15, 0.5]$, for subsampling factors 2 and 4 (respective spectrum binning $32 \times 32$ and $16 \times 16$). The subsampling was done through a boxcar bandpass function. No blur, no noise. This helps check the match between real scenes and the proposed fractal image model, shown as a straight dashed line. One should expect a similar behavior at the two different scales for a perfectly fractal image, in addition to the good match.

### TABLE 7.10: Summary of the experimental results for images of Matangi and Hellinko, subsampled by factors 2 and 4 using a Boxcar bandpass function. For each setting (labeled from a to h), we show the 3 true parameter values and MTF values at 3 points in the frequency space, followed by the 6 corresponding estimated values. Then we show the normalized $\chi^2$ residual (defined in subsection 7.5.2) and the rejected outlier rates, denoted by $k$ and $\text{rej}$. The last 3 columns display the MTF errors (estimated-true) at the 3 chosen points in the frequency space.
### Results

<table>
<thead>
<tr>
<th>#</th>
<th>True $\sigma^2$, $\alpha_1$, $\alpha_2$, MTF (x4)</th>
<th>Est $\sigma^2$, $\alpha_1$, $\alpha_2$, MTF (x4)</th>
<th>Err MTF (x4)</th>
<th>k, rej</th>
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<tbody>
<tr>
<td>a</td>
<td>1.00 10.0 10.0 0.48 0.48 0.23</td>
<td>1.08 10.0 9.8 0.48 0.48 0.23</td>
<td>0.16 29%</td>
<td>0.00 &lt;0.01 =0.00</td>
</tr>
<tr>
<td>b</td>
<td>1.00 10.0 30.0 0.48 0.48 0.23</td>
<td>1.08 10.0 30.0 0.48 0.48 0.23</td>
<td>0.16 29%</td>
<td>0.00 &lt;0.01 &lt;0.01</td>
</tr>
<tr>
<td>c</td>
<td>1.00 30.0 30.0 0.48 0.48 0.23</td>
<td>1.08 28.8 28.8 0.48 0.48 0.23</td>
<td>0.16 29%</td>
<td>0.00 &lt;0.01 &lt;0.01</td>
</tr>
<tr>
<td>d</td>
<td>1.00 30.0 30.0 0.48 0.48 0.23</td>
<td>1.08 30.0 30.0 0.48 0.48 0.23</td>
<td>0.16 29%</td>
<td>0.00 &lt;0.01 &lt;0.01</td>
</tr>
<tr>
<td>e</td>
<td>1.00 10.0 10.0 0.48 0.48 0.23</td>
<td>0.96 9.9 9.9 0.48 0.48 0.23</td>
<td>0.16 29%</td>
<td>0.00 &lt;0.01 &lt;0.01</td>
</tr>
<tr>
<td>f</td>
<td>1.00 30.0 30.0 0.48 0.48 0.23</td>
<td>0.96 30.0 30.0 0.48 0.48 0.23</td>
<td>0.16 29%</td>
<td>0.00 &lt;0.01 &lt;0.01</td>
</tr>
<tr>
<td>g</td>
<td>1.00 30.0 30.0 0.48 0.48 0.23</td>
<td>0.96 30.0 30.0 0.48 0.48 0.23</td>
<td>0.16 29%</td>
<td>0.00 &lt;0.01 &lt;0.01</td>
</tr>
<tr>
<td>h</td>
<td>1.00 30.0 30.0 0.48 0.48 0.23</td>
<td>0.96 30.0 30.0 0.48 0.48 0.23</td>
<td>0.16 29%</td>
<td>0.00 &lt;0.01 &lt;0.01</td>
</tr>
</tbody>
</table>

**Gooseneck 1024 × 1024 (bin size 32 × 32)**

| a | 1.00 10.0 10.0 0.48 0.48 0.23 | 1.08 11.5 8.7 0.44 0.52 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| b | 1.00 10.0 30.0 0.48 0.48 0.23 | 1.08 10.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| c | 1.00 30.0 30.0 0.48 0.48 0.23 | 1.08 28.8 28.8 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| d | 1.00 30.0 30.0 0.48 0.48 0.23 | 1.08 30.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| e | 1.00 10.0 10.0 0.48 0.48 0.23 | 0.96 9.9 9.9 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| f | 1.00 30.0 30.0 0.48 0.48 0.23 | 0.96 30.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| g | 1.00 30.0 30.0 0.48 0.48 0.23 | 0.96 30.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| h | 1.00 30.0 30.0 0.48 0.48 0.23 | 0.96 30.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |

**Stonehenge 1024 × 1024 (bin size 32 × 32)**

| a | 1.00 10.0 10.0 0.48 0.48 0.23 | 1.08 10.0 10.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| b | 1.00 10.0 30.0 0.48 0.48 0.23 | 1.08 10.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| c | 1.00 30.0 30.0 0.48 0.48 0.23 | 1.08 30.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| d | 1.00 30.0 30.0 0.48 0.48 0.23 | 1.08 30.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| e | 1.00 10.0 10.0 0.48 0.48 0.23 | 0.96 9.9 9.9 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| f | 1.00 30.0 30.0 0.48 0.48 0.23 | 0.96 30.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| g | 1.00 30.0 30.0 0.48 0.48 0.23 | 0.96 30.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |
| h | 1.00 30.0 30.0 0.48 0.48 0.23 | 0.96 30.0 30.0 0.48 0.48 0.23 | 0.16 29% | 0.00 <0.01 <0.01 |

**FIGURE 7.10:** Summary of the experimental results for images of Gooseneck and Stonehenge, subsampled by factors 2 and 4 using a Boxcar bandpass function. For each setting (labeled from a to h), we show the 3 true parameter values and MTF values at 3 points in the frequency space, followed by the 6 corresponding estimated values. Then we show the normalized $\chi^2$ residual (defined in subsection 7.5.2) and the rejected outlier rates, denoted by k and rej. The last 3 columns display the MTF errors (estimated–true) at the 3 chosen points in the frequency space.

### 7.6 Conclusion

We proposed a new framework for the estimation of the noise and transfer function parameters from any remote sensing image. This involves a fractal parametric model of the underlying natural scene and a parametric MTF that takes into account the optics and the sensor. The stability of the estimation method is due to the marginalization of the image model parameters.

We first proposed an initial algorithm that was successfully applied to both simulated and real data, and patented. Recently, we developed an optimized technique based on a simplified model such that the algorithmic complexity is mainly due to one FFT for large images, not to the estimation itself.

Good quality estimates were obtained in an entirely unsupervised way. As shown experimentally, random images (from urban to natural areas) could be used, without specific features such as targets or coast lines, as long as their power spectrum content is sufficient. The developed framework is flexible enough to allow different kinds of instruments to be modeled, as long as optics and sensor blur can be accurately described by parametric models.

In the future, field experiments shall be carried out in order to validate this approach, through the use of real blurred and noisy remote sensing images, as well as precise means of determining the true values of the blur parameters. A performance comparison should also be performed to help determine the net benefit of the proposed technique against some recent ones that have been applied to the exact same type of data, but based on different formalisms and assumptions.

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### References


References

cessing Magazine, 13(3), May 1996.


Bayesian estimation of blur and noise in remote sensing imaging


References


